

Lorentz covariant nucleon self-energy decomposition of the nuclear symmetry energy

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Using the Hugenholtz-Van Hove theorem, we derive analytical expressions for the nuclear symmetry energy $E_{\text{sym}}(\rho)$ and its density slope $L(\rho)$ in terms of the Lorentz covariant nucleon self-energies in isospin asymmetric nuclear matter. These general expressions are useful for determining the density dependence of the symmetry energy and understanding the Lorentz structure and the microscopic origin of the symmetry energy in relativistic covariant formalism. As an example, we analyze the Lorentz covariant nucleon self-energy decomposition of $E_{\text{sym}}(\rho)$ and $L(\rho)$ and derive the corresponding analytical expressions within the nonlinear σ - ω - ρ - δ relativistic mean field model.

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I. INTRODUCTION

In the current research of nuclear physics and astrophysics, there is of great interest to study the density dependence of the nuclear symmetry energy $E_{\text{sym}}(\rho)$ that essentially characterizes the isospin dependent part of the equation of state (EOS) of asymmetric nuclear matter. The exact knowledge on the symmetry energy is important for understanding not only many problems in nuclear physics, but also many critical topics in astrophysics [1–6] as well as some interesting issues regarding possible new physics beyond the standard model [7–9]. During the last decade, although significant progress has been made both experimentally and theoretically on constraining the density dependence of the symmetry energy [5, 6, 10–13] (see, e.g., Refs. [14–18] for review of recent progress), large uncertainties on $E_{\text{sym}}(\rho)$ still exist, especially its super-normal density behavior remains elusive and largely controversial [19–22]. To reduce the uncertainties of the constraints on $E_{\text{sym}}(\rho)$ is thus of critical importance and remains a big challenge in the community, and this provides a strong motivation for studying isospin nuclear physics in radioactive nuclei at the new/planning rare isotope beam facilities around the world, such as CSR/Lanzhou and BRIF-II/Beijing in China, RIBF/RIKEN in Japan, SPIRAL2/GANIL in France, FAIR/GSI in Germany, FRIB/NSCL in USA, SPES/LNL in Italy, and KoRIA in Korea.

Theoretically, the uncertainties of the constraints on $E_{\text{sym}}(\rho)$ are mainly due to the lack of knowledge about the isospin dependence of in-medium nuclear effective interactions and the limitations in the techniques for solving the nuclear many body problem. Very recently, it has been proposed that it is very useful to directly decompose $E_{\text{sym}}(\rho)$ in terms of some relevant parts of the commonly used underlying nuclear effective interaction in nuclear medium [14, 23, 24]. Based on the Hugenholtz-Van Hove

(HVV) theorem, indeed, the $E_{\text{sym}}(\rho)$ can be decomposed analytically in terms of the single-nucleon potential in asymmetric nuclear matter and the resulting expressions are quite general and independent of the detailed nature of the nucleon interactions, providing an important and physically more transparent approach to extract information on the symmetry energy from the isospin dependence of strong interaction in nuclear medium and understand why the symmetry energy predicted from various models is so uncertain [22, 24]. In these works, the decomposition of $E_{\text{sym}}(\rho)$ is based on non-relativistic framework. It is thus of great interest to explore more general decomposition within relativistic covariant framework, which is the main motivation of the present work.

II. COVARIANT SELF-ENERGY DECOMPOSITION OF $E_{\text{sym}}(\rho)$ AND $L(\rho)$

The relativistic covariant formulation has made great success during the last decades in understanding many nuclear phenomena [25–27]. In particular, the microscopic relativistic covariant Dirac-Brueckner-Hartree-Fock (DBHF) approach [28–33] has achieved impressive success in describing the saturation properties of nuclear matter without any need to introduce a three-nucleon force required in the microscopic non-relativistic BHF calculations (see, e.g., Refs. [34, 35]). It has been argued that in non-relativistic calculations the three-nucleon forces must be introduced to mimic the variation of the Dirac spinors in the nuclear medium contained in relativistic covariant approach [32]. In addition, the Lorentz covariant decomposition of the nuclear mean field potential has been shown to be very important for understanding the dynamics in heavy ion collisions at relativistic energies [36–38]. These features imply that the Lorentz covariance could be important for understanding the higher energy/density nuclear phenomena, e.g., the high density behaviors of the symmetry energy.

Owing to the translational, rotational and time-reversal invariance, parity conservation, and hermiticity, the Lorentz covariant nucleon self-energy in the rest

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frame of asymmetric nuclear matter with baryon density $\rho = \rho_n + \rho_p$ and isospin asymmetry $\alpha = (\rho_n - \rho_p)/\rho$ can be written generally as [25, 29, 39–41],

$$\begin{aligned}\Sigma^J(\rho, \alpha, |\mathbf{k}|) &= \Sigma_S^J(\rho, \alpha, |\mathbf{k}|) - \gamma_\mu \Sigma^{\mu,J}(\rho, \alpha, |\mathbf{k}|) \\ &= \Sigma_S^J(\rho, \alpha, |\mathbf{k}|) + \gamma^0 \Sigma_V^J(\rho, \alpha, |\mathbf{k}|) \\ &\quad + \gamma \cdot \mathbf{k}^0 \Sigma_K^J(\rho, \alpha, |\mathbf{k}|),\end{aligned}\quad (1)$$

where $J = n$ or p is isospin index; $\Sigma_S^J(\rho, \alpha, |\mathbf{k}|)$ is the scalar self-energy, $\Sigma_V^J(\rho, \alpha, |\mathbf{k}|) \equiv -\Sigma^{0,J}(\rho, \alpha, |\mathbf{k}|)$ and $\Sigma_K^J(\rho, \alpha, |\mathbf{k}|)$ are, respectively, the zeroth component and the space component of the vector self-energy $\Sigma^{\mu,J}(\rho, \alpha, |\mathbf{k}|)$, and they all generally depend on ρ , α and the magnitude of the nucleon momentum $|\mathbf{k}|$ (Minkowski metric is $g_{\mu\nu} = (+, -, -, -)$); $\mathbf{k}^0 = \mathbf{k}/|\mathbf{k}|$ is the unit vector of momentum \mathbf{k} . A proof of Eq. (1) can be found in Ref. [25]. Accordingly, the single-nucleon energy can be expressed as [25, 29, 39–41]

$$\mathcal{E}^J(\rho, \alpha, |\mathbf{k}|) = \mathcal{E}^{*J}(\rho, \alpha, |\mathbf{k}|) + \Sigma_V^J(\rho, \alpha, |\mathbf{k}|), \quad (2)$$

with

$$\mathcal{E}^{*J}(\rho, \alpha, |\mathbf{k}|) = \sqrt{[\mathbf{k}^{*J}(\rho, \alpha, |\mathbf{k}|)]^2 + [M_J^*(\rho, \alpha, |\mathbf{k}|)]^2}, \quad (3)$$

where the nucleon effective (Dirac) mass $M_J^*(\rho, \alpha, |\mathbf{k}|)$ and effective momentum $\mathbf{k}^{*J}(\rho, \alpha, |\mathbf{k}|)$ are defined, respectively, as

$$M_J^*(\rho, \alpha, |\mathbf{k}|) = M + \Sigma_S^J(\rho, \alpha, |\mathbf{k}|), \quad (4)$$

$$\mathbf{k}^{*J}(\rho, \alpha, |\mathbf{k}|) = \mathbf{k} + \mathbf{k}^0 \Sigma_K^J(\rho, \alpha, |\mathbf{k}|), \quad (5)$$

with M being the nucleon mass.

Due to the exchange symmetry between protons and neutrons in nuclear matter, the EOS of asymmetric nuclear matter, defined by its binding energy per nucleon, can be expanded as a power series of even-order terms in α as

$$E(\rho, \alpha) \simeq E_0(\rho) + E_{\text{sym}}(\rho)\alpha^2 + \mathcal{O}(\alpha^4), \quad (6)$$

where $E_0(\rho) = E(\rho, \alpha = 0)$ is the EOS of symmetric nuclear matter, and the symmetry energy is expressed as

$$E_{\text{sym}}(\rho) = \frac{1}{2} \frac{\partial^2 E(\rho, \alpha)}{\partial \alpha^2} \Big|_{\alpha=0}. \quad (7)$$

Around the nuclear matter saturation density ρ_0 , the symmetry energy can be expanded as

$$E_{\text{sym}}(\rho) \simeq E_{\text{sym}}(\rho_0) + L\chi + \mathcal{O}(\chi^2), \quad (8)$$

where $\chi = (\rho - \rho_0)/3\rho_0$ is a dimensionless variable, and $L = L(\rho_0)$ is the density slope parameter of the symmetry energy at ρ_0 and thus carries important information on the symmetry energy at both high and low densities. Generally, the slope parameter of the symmetry energy at arbitrary density ρ is defined as

$$L(\rho) = 3\rho \frac{dE_{\text{sym}}(\rho)}{d\rho}. \quad (9)$$

According to the HVH theorem [42, 43], the nucleon chemical potential in asymmetric nuclear matter should be equal to its Fermi energy (the single-nucleon energy at Fermi surface), i.e.,

$$\mathcal{E}_F^J(\rho, \alpha, k_F^J) = \frac{\partial[\rho E(\rho, \alpha)]}{\partial \rho_J} + M, \quad (10)$$

where $\mathcal{E}_F^J(\rho, \alpha, k_F^J) \equiv \mathcal{E}^J(\rho, \alpha, k_F^J)$ is the nucleon Fermi energy, and $k_F^J = k_F(1 + \tau_3^J \alpha)^{1/3}$ (we assume $\tau_3^n = 1$ and $\tau_3^p = -1$ in this work) is the nucleon Fermi momentum with $k_F = (3\pi^2 \rho/2)^{1/3}$ being the Fermi momentum in symmetric nuclear matter at density ρ . It should be noted that the HVH theorem is independent of the detailed nature of the interactions used and is valid for any interacting self-bound infinite Fermi system, such as infinite nuclear matter [41–43].

Expanding $E(\rho, \alpha)$ as a power series of even-order terms in α on the right-hand side of Eq. (10), we can obtain

$$\begin{aligned}\sum_{J=p,n} \tau_3^J \mathcal{E}_F^J(\rho, \alpha, k_F^J) &= \sum_{i=1} 4i E_{\text{sym},2i}(\rho) \alpha^{2i-1}, \\ \sum_{J=p,n} \mathcal{E}_F^J(\rho, \alpha, k_F^J) &= 2 \frac{\partial[\rho E_0(\rho)]}{\partial \rho} + 2M \\ &\quad + 2 \frac{\partial}{\partial \rho} \sum_{i=1} \rho E_{\text{sym},2i}(\rho) \alpha^{2i},\end{aligned}\quad (11)$$

where $E_{\text{sym},2i}(\rho) \equiv [1/(2i)!][\partial^{2i} E(\rho, \alpha)/\partial \alpha^{2i}]_{\alpha=0}$ are the symmetry energies of different orders and particularly we have $E_{\text{sym},2}(\rho) \equiv E_{\text{sym}}(\rho)$. Furthermore, expanding $\mathcal{E}_F^J(\rho, \alpha, k_F^J)$ as a power series of α on the left-hand side of Eq. (11) and Eq. (12), and comparing the coefficients of the first-order α terms on both left- and right-hand sides of Eq. (11), we then obtain

$$E_{\text{sym}}(\rho) = \frac{1}{4} \frac{d}{d\alpha} \left[\sum_{J=p,n} \tau_3^J \mathcal{E}_F^J(\rho, \alpha, k_F^J) \right] \Big|_{\alpha=0}, \quad (13)$$

while comparing the coefficients of second-order α terms on both sides of Eq. (12) leads to the following expression:

$$L(\rho) = \frac{3}{4} \frac{d^2}{d\alpha^2} \left[\sum_{J=p,n} \mathcal{E}_F^J(\rho, \alpha, k_F^J) \right] \Big|_{\alpha=0} + 3E_{\text{sym}}(\rho). \quad (14)$$

Substituting Eq. (2) into Eq. (13), we can obtain

$$\begin{aligned}E_{\text{sym}}(\rho) &= E_{\text{sym}}^{\text{kin}}(\rho) \\ &\quad + E_{\text{sym}}^{0,\text{mom},K}(\rho) + E_{\text{sym}}^{0,\text{mom},S}(\rho) + E_{\text{sym}}^{0,\text{mom},V}(\rho) \\ &\quad + E_{\text{sym}}^{1\text{st},K}(\rho) + E_{\text{sym}}^{1\text{st},S}(\rho) + E_{\text{sym}}^{1\text{st},V}(\rho),\end{aligned}\quad (15)$$

where $E_{\text{sym}}^{\text{kin}}(\rho)$, $E_{\text{sym}}^{0,\text{mom},\mathcal{O}}(\rho)$ and $E_{\text{sym}}^{1\text{st},\mathcal{O}}(\rho)$ (here \mathcal{O} denotes K, S or V) represent, respectively, the contributions from the kinetic part, the momentum dependence

of the nucleon self-energies in symmetric nuclear matter and the first-order symmetry self-energies, and they can be expressed analytically as

$$E_{\text{sym}}^{\text{kin}}(\rho) = \frac{k_F k_F^*(\rho)}{6\mathcal{E}_F^*(\rho)}, \quad (16)$$

$$E_{\text{sym}}^{0,\text{mom},\text{K}}(\rho) = \frac{k_F k_F^*(\rho)}{6\mathcal{E}_F^*(\rho)} \left. \frac{\partial \Sigma_K^0(\rho, |\mathbf{k}|)}{\partial |\mathbf{k}|} \right|_{|\mathbf{k}|=k_F}, \quad (17)$$

$$E_{\text{sym}}^{0,\text{mom},\text{S}}(\rho) = \frac{k_F M_0^*(\rho)}{6\mathcal{E}_F^*(\rho)} \left. \frac{\partial \Sigma_S^0(\rho, |\mathbf{k}|)}{\partial |\mathbf{k}|} \right|_{|\mathbf{k}|=k_F}, \quad (18)$$

$$E_{\text{sym}}^{0,\text{mom},\text{V}}(\rho) = \frac{k_F}{6} \left. \frac{\partial \Sigma_V^0(\rho, |\mathbf{k}|)}{\partial |\mathbf{k}|} \right|_{|\mathbf{k}|=k_F}, \quad (19)$$

$$E_{\text{sym}}^{1\text{st},\text{K}}(\rho) = \frac{1}{2} \frac{k_F^*(\rho) \Sigma_K^{\text{sym},1}(\rho, |\mathbf{k}| = k_F)}{\mathcal{E}_F^*(\rho)}, \quad (20)$$

$$E_{\text{sym}}^{1\text{st},\text{S}}(\rho) = \frac{1}{2} \frac{M_0^*(\rho) \Sigma_S^{\text{sym},1}(\rho, |\mathbf{k}| = k_F)}{\mathcal{E}_F^*(\rho)}, \quad (21)$$

$$E_{\text{sym}}^{1\text{st},\text{V}}(\rho) = \frac{1}{2} \Sigma_V^{\text{sym},1}(\rho, |\mathbf{k}| = k_F), \quad (22)$$

where $k_F^*(\rho) = k_F + \Sigma_K^0(\rho, k_F)$, $M_0^*(\rho) = M + \Sigma_S^0(\rho, k_F)$, $\mathcal{E}_F^*(\rho) = (k_F^{*2} + M_0^{*2})^{1/2}$, $\Sigma_K^0(\rho, |\mathbf{k}|) = \Sigma_K^J(\rho, \alpha = 0, |\mathbf{k}|)$, $\Sigma_S^0(\rho, |\mathbf{k}|) = \Sigma_S^J(\rho, \alpha = 0, |\mathbf{k}|)$, $\Sigma_V^0(\rho, |\mathbf{k}|) = \Sigma_V^J(\rho, \alpha = 0, |\mathbf{k}|)$, and the i -th order symmetry self-energy is defined as (here $\mathcal{O} = \text{K, S, V}$)

$$\Sigma_{\mathcal{O}}^{\text{sym},i}(\rho, |\mathbf{k}|) = \frac{1}{i!} \frac{\partial^i}{\partial \alpha^i} \left[\sum_{J=\text{p,n}} \frac{\tau_3^{Ji} \Sigma_{\mathcal{O}}^J(\rho, \alpha, |\mathbf{k}|)}{2} \right] \Big|_{\alpha=0}. \quad (23)$$

Furthermore, Eq. (15) can be rewritten as

$$E_{\text{sym}}(\rho) = \frac{|\mathbf{k}|^2}{6M_{0,\text{Lan}}^*(\rho, |\mathbf{k}|)} \Big|_{|\mathbf{k}|=k_F} + E_{\text{sym}}^{1\text{st},\text{K}}(\rho) + E_{\text{sym}}^{1\text{st},\text{S}}(\rho) + E_{\text{sym}}^{1\text{st},\text{V}}(\rho), \quad (24)$$

where $M_{0,\text{Lan}}^*(\rho, |\mathbf{k}|)$ is the nucleon Landau mass in symmetric nuclear matter, i.e., $M_{0,\text{Lan}}^*(\rho, |\mathbf{k}|) = |\mathbf{k}| [d|\mathbf{k}|/d\mathcal{E}^0(\rho, |\mathbf{k}|)]^{-1}$ (see, e.g., Ref. [44]) with $\mathcal{E}^0(\rho, |\mathbf{k}|) = \mathcal{E}^J(\rho, \alpha = 0, |\mathbf{k}|)$, and one can easily verify the relation $k_F^2/6M_{0,\text{Lan}}^*(\rho, k_F) = E_{\text{sym}}^{\text{kin}}(\rho) + E_{\text{sym}}^{0,\text{mom},\text{K}}(\rho) + E_{\text{sym}}^{0,\text{mom},\text{S}}(\rho) + E_{\text{sym}}^{0,\text{mom},\text{V}}(\rho)$. In this way, we have decomposed analytically the symmetry energy $E_{\text{sym}}(\rho)$ in terms of the Lorentz covariant nucleon self-energies in asymmetric nuclear matter.

Similarly, by substituting Eq. (2) into Eq. (14), the slope parameter $L(\rho)$ can be decomposed as

$$L(\rho) = L^{\text{kin}}(\rho) + L^{\text{mom}}(\rho) + L^{1\text{st}}(\rho) + L^{\text{cross}}(\rho) + L^{2\text{nd}}(\rho), \quad (25)$$

with

$$L^{\text{kin}}(\rho) = \frac{k_F k_F^*}{6\mathcal{E}_F^*} + \frac{k_F^2 M_0^{*2}}{6\mathcal{E}_F^{*3}}, \quad (26)$$

$$\begin{aligned} L^{\text{mom}}(\rho) &= \frac{k_F^2 M_0^{*2}}{3\mathcal{E}_F^{*2}} \left. \frac{\partial \Sigma_K^0}{\partial |\mathbf{k}|} \right|_{|\mathbf{k}|=k_F} \\ &+ \frac{k_F^2}{6} \left[\frac{k_F^*}{\mathcal{E}_F^*} \frac{\partial^2 \Sigma_K^0}{\partial |\mathbf{k}|^2} + \frac{M_0^*}{\mathcal{E}_F^*} \frac{\partial^2 \Sigma_S^0}{\partial |\mathbf{k}|^2} + \frac{\partial^2 \Sigma_V^0}{\partial |\mathbf{k}|^2} \right]_{|\mathbf{k}|=k_F} \\ &+ \frac{k_F}{6} \left[\frac{k_F^*}{\mathcal{E}_F^*} \frac{\partial \Sigma_K^0}{\partial |\mathbf{k}|} + \frac{M_0^*}{\mathcal{E}_F^*} \frac{\partial \Sigma_S^0}{\partial |\mathbf{k}|} + \frac{\partial \Sigma_V^0}{\partial |\mathbf{k}|} \right]_{|\mathbf{k}|=k_F} \\ &+ \frac{k_F^2}{6\mathcal{E}_F^{*3}} \left[M_0^{*2} \left(\frac{\partial \Sigma_K^0}{\partial |\mathbf{k}|} \right)^2 + k_F^{*2} \left(\frac{\partial \Sigma_S^0}{\partial |\mathbf{k}|} \right)^2 \right]_{|\mathbf{k}|=k_F} \\ &- \frac{k_F^2 k_F^* M_0^*}{3\mathcal{E}_F^{*3}} \left[\frac{\partial \Sigma_S^0}{\partial |\mathbf{k}|} \left(1 + \frac{\partial \Sigma_K^0}{\partial |\mathbf{k}|} \right) \right]_{|\mathbf{k}|=k_F}, \end{aligned} \quad (27)$$

$$\begin{aligned} L^{1\text{st}}(\rho) &= \frac{3}{2\mathcal{E}_F^{*3}} \left[M_0^* \Sigma_K^{\text{sym},1} - k_F^* \Sigma_S^{\text{sym},1} \right]^2 \\ &+ \frac{3}{2} \left[\frac{k_F^*}{\mathcal{E}_F^*} \Sigma_K^{\text{sym},1} + \frac{M_0^*}{\mathcal{E}_F^*} \Sigma_S^{\text{sym},1} + \Sigma_V^{\text{sym},1} \right] \\ &+ \frac{k_F M_0^{*2} \Sigma_K^{\text{sym},1}}{\mathcal{E}_F^{*3}} - \frac{k_F k_F^* M_0^* \Sigma_S^{\text{sym},1}}{\mathcal{E}_F^{*3}}, \end{aligned} \quad (28)$$

$$\begin{aligned} L^{\text{cross}}(\rho) &= k_F \left[\frac{k_F^*}{\mathcal{E}_F^*} \frac{\partial \Sigma_K^{\text{sym},1}}{\partial |\mathbf{k}|} + \frac{M_0^*}{\mathcal{E}_F^*} \frac{\partial \Sigma_S^{\text{sym},1}}{\partial |\mathbf{k}|} + \frac{\partial \Sigma_V^{\text{sym},1}}{\partial |\mathbf{k}|} \right]_{|\mathbf{k}|=k_F} \\ &- \frac{k_F \Sigma_K^{\text{sym},1}}{\mathcal{E}_F^*} \left[\frac{k_F^{*2}}{\mathcal{E}_F^{*2}} \left(\frac{\partial \Sigma_K^0}{\partial |\mathbf{k}|} + \frac{M_0^*}{k_F^*} \frac{\partial \Sigma_S^0}{\partial |\mathbf{k}|} \right) - \frac{\partial \Sigma_K^0}{\partial |\mathbf{k}|} \right]_{|\mathbf{k}|=k_F} \\ &- \frac{k_F \Sigma_S^{\text{sym},1}}{\mathcal{E}_F^*} \left[\frac{M_0^{*2}}{\mathcal{E}_F^{*2}} \left(\frac{k_F^*}{M_0^*} \frac{\partial \Sigma_K^0}{\partial |\mathbf{k}|} + \frac{\partial \Sigma_S^0}{\partial |\mathbf{k}|} \right) - \frac{\partial \Sigma_S^0}{\partial |\mathbf{k}|} \right]_{|\mathbf{k}|=k_F}, \end{aligned} \quad (29)$$

$$L^{2\text{nd}}(\rho) = 3 \left[\frac{k_F^*}{\mathcal{E}_F^*} \Sigma_K^{\text{sym},2} + \frac{M_0^*}{\mathcal{E}_F^*} \Sigma_S^{\text{sym},2} + \Sigma_V^{\text{sym},2} \right]. \quad (30)$$

On the right-hand side of Eqs. (26)-(30), the density and momentum dependence have been suppressed with $\Sigma_{\mathcal{O}}^{\text{sym},i} = \Sigma_{\mathcal{O}}^{\text{sym},i}(\rho, |\mathbf{k}| = k_F)$ ($\mathcal{O} = \text{K, S, V}$). Eq. (15) (or (24)) and Eq. (25) are two main results of this work.

III. APPLICATION TO THE NONLINEAR σ - ω - ρ - δ RMF MODEL

The nucleon self-energies can be calculated theoretically from a certain relativistic covariant approach or extracted experimentally (around ρ_0) from the Dirac phenomenology of nucleon-nucleus scattering. The Lorentz covariant nucleon self-energy decompositions of $E_{\text{sym}}(\rho)$ in Eq. (15) (or (24)) and $L(\rho)$ in Eq. (25) are general and they are useful for determining the density dependence of the symmetry energy and understanding its Lorentz structure and the microscopic origin. As an example, we consider here the nonlinear σ - ω - ρ - δ relativistic mean field (RMF) model which is based on effective interaction Lagrangians involving nucleon and meson fields, and has been widely discussed in the literature (see, e.g., Ref. [44]). A very useful feature of this model is that

the nucleon self-energies in asymmetric nuclear matter can be obtained analytically and this makes our analysis physically transparent. The Lagrangian density of the nonlinear σ - ω - ρ - δ RMF model can be expressed as (see, e.g., Ref. [44]):

$$\begin{aligned}\mathcal{L} = & \bar{\psi} [\gamma_\mu (i\partial^\mu - g_\omega \omega^\mu) - (M - g_\sigma \sigma)] \psi \\ & + \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu \\ & - \frac{1}{3} b_\sigma M (g_\sigma \sigma)^3 - \frac{1}{4} c_\sigma (g_\sigma \sigma)^4 + \frac{1}{4} c_\omega (g_\omega^2 \omega_\mu \omega^\mu)^2 \\ & + \frac{1}{2} (\partial_\mu \vec{\delta} \cdot \partial^\mu \vec{\delta} - m_\delta^2 \vec{\delta}^2) + \frac{1}{2} m_\rho^2 \vec{\rho}_\mu \cdot \vec{\rho}^\mu - \frac{1}{4} \vec{G}_{\mu\nu} \cdot \vec{G}^{\mu\nu} \\ & + \frac{1}{2} (g_\rho^2 \vec{\rho}_\mu \cdot \vec{\rho}^\mu) (\Lambda_S g_\sigma^2 \sigma^2 + \Lambda_V g_\omega^2 \omega_\mu \omega^\mu) \\ & - g_\rho \vec{\rho}_\mu \cdot \bar{\psi} \gamma^\mu \vec{\tau}^J \psi + g_\delta \vec{\delta} \cdot \bar{\psi} \vec{\tau}^J \psi,\end{aligned}\quad (31)$$

where $F_{\mu\nu} \equiv \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$ and $\vec{G}_{\mu\nu} \equiv \partial_\mu \vec{\rho}_\nu - \partial_\nu \vec{\rho}_\mu$ are strength tensors of ω field and ρ field, respectively while ψ , σ , ω_μ , $\vec{\rho}_\mu$ and $\vec{\delta}$ are nucleon field, isoscalar-scalar field, isoscalar-vector field, isovector-vector and isovector-scalar field, respectively, and the arrows denote the isospin vector. Λ_S and Λ_V are two cross-coupling constants for varying the density dependence of $E_{\text{sym}}(\rho)$, and m_σ , m_ω , m_ρ , and m_δ are masses of mesons.

In the RMF model, meson fields are replaced by their expectation values, i.e., $\bar{\sigma} \rightarrow \sigma$, $\bar{\omega}_0 \rightarrow \omega_\mu$, $\bar{\rho}_0^{(3)} \rightarrow \vec{\rho}_\mu$, where the subscript “0” denotes the zeroth component of the four-vector while the superscript “(3)” denotes the third component of isospin. Furthermore, the space-like self-energy $\Sigma_K^J(\rho, \delta, |\mathbf{k}|)$ vanishes (due to the Hartree approximation in the RMF model) while the scalar and time-like self-energies are momentum independent, i.e.,

$$\Sigma_S^J(\rho, \alpha) = -g_\sigma \bar{\sigma} + \tau_3^J g_\delta \bar{\delta}^{(3)}, \quad (32)$$

$$\Sigma_V^J(\rho, \alpha) = g_\omega \bar{\omega}_0 - \tau_3^J g_\rho \bar{\rho}_0^{(3)}. \quad (33)$$

The symmetry energy then can be decomposed as

$$\begin{aligned}E_{\text{sym}}(\rho) = & E_{\text{sym}}^{\text{kin}}(\rho) + E_{\text{sym}}^{\text{1st},S}(\rho) + E_{\text{sym}}^{\text{1st},V}(\rho) \\ = & \frac{k_F^2}{6\mathcal{E}_F^*} + \frac{1}{2} \frac{M_0^* \Sigma_S^{\text{sym},1}(\rho)}{\mathcal{E}_F^*} + \frac{1}{2} \Sigma_V^{\text{sym},1}(\rho),\end{aligned}\quad (34)$$

where the (1st-order) symmetry self-energies are

$$\Sigma_S^{\text{sym},1}(\rho) = -\frac{g_\delta^2 M_0^* \rho}{\mathcal{E}_F^* Q_\delta}, \quad (35)$$

$$\Sigma_V^{\text{sym},1}(\rho) = +\frac{g_\rho^2 \rho}{Q_\rho}, \quad (36)$$

with $Q_\delta = m_\delta^2 + 3g_\delta^2(\rho_S/M_0^* - \rho/\mathcal{E}_F^*)$, ρ_S being the scalar density, and $Q_\rho = m_\rho^2 + \Lambda_S g_\sigma^2 g_\rho^2 \bar{\sigma}^2 + \Lambda_V g_\omega^2 g_\rho^2 \bar{\omega}_0^2$. We note that the above analytical expression for $E_{\text{sym}}(\rho)$ is exactly the same as the one obtained from the normal approach (see, e.g., Ref. [44]). Similarly, the slope parameter $L(\rho)$ can be decomposed as

$$L(\rho) = L^{\text{kin}}(\rho) + L^{\text{1st}}(\rho) + L^{\text{2nd}}(\rho), \quad (37)$$

with

$$L^{\text{kin}}(\rho) = \frac{k_F^2 (\mathcal{E}_F^{*2} + M_0^{*2})}{6\mathcal{E}_F^{*3}}, \quad (38)$$

$$\begin{aligned}L^{\text{1st}}(\rho) = & \frac{3}{2} \left[\frac{M_0^* \Sigma_S^{\text{sym},1}(\rho)}{\mathcal{E}_F^*} + \Sigma_V^{\text{sym},1}(\rho) \right] \\ & + \frac{3k_F^2}{2\mathcal{E}_F^{*3}} \left[\Sigma_S^{\text{sym},1}(\rho) \right]^2 - \frac{M_0^* k_F^2 \Sigma_S^{\text{sym},1}(\rho)}{\mathcal{E}_F^{*3}},\end{aligned}\quad (39)$$

$$L^{\text{2nd}}(\rho) = 3 \left[\frac{M_0^* \Sigma_S^{\text{sym},2}(\rho)}{\mathcal{E}_F^*} + \Sigma_V^{\text{sym},2}(\rho) \right], \quad (40)$$

where the 2nd-order symmetry self-energies are

$$\begin{aligned}\Sigma_S^{\text{sym},2}(\rho) = & -\frac{g_\sigma}{2Q_\sigma} \left(\frac{g_\delta^2 M_0^* \rho \Gamma}{\mathcal{E}_F^{*2} Q_\delta} - \frac{2g_\delta^2 M_0^* k_F^2 \rho^2}{\mathcal{E}_F^{*4} Q_\delta} \right. \\ & \left. - \frac{g_\sigma M_0^* k_F^2 \rho}{3\mathcal{E}_F^{*3}} + \frac{2g_\sigma^2 g_\rho^4 \Lambda_S \bar{\sigma} \rho^2}{Q_\rho^2} \right),\end{aligned}\quad (41)$$

$$\Sigma_V^{\text{sym},2}(\rho) = -\frac{g_\omega^3 g_\rho^4 \Lambda_V \bar{\omega}_0 \rho^2}{Q_\omega Q_\rho^2}, \quad (42)$$

with $Q_\sigma = m_\sigma^2 + g_\sigma^2(3\rho_S/M_0^* - 3\rho/\mathcal{E}_F^*) + 2b_\sigma M g_\sigma^3 \bar{\sigma} + 3c_\sigma g_\sigma^4 \bar{\sigma}^2$, $Q_\omega = m_\omega^2 + 3c_\omega g_\omega^4 \bar{\omega}_0^2$ and $\Gamma = 3g_\sigma g_\delta^2(2\rho_S/M_0^{*2} - 3\rho/M_0^* \mathcal{E}_F^* + M_0^* \rho/\mathcal{E}_F^{*3})$. To the best of our knowledge, the above formulas give, for the first time, the analytical expression of the slope parameter $L(\rho)$ in the nonlinear RMF model. The above analytical expressions of $E_{\text{sym}}(\rho)$ and $L(\rho)$ can be easily generalized to the case of the density dependent RMF model that has similar isospin structure as the nonlinear RMF model [44]. It should be mentioned that these analytical expressions for $E_{\text{sym}}(\rho)$ and $L(\rho)$ are very useful for determining the isovector parameters in the RMF model by fitting the empirical properties of asymmetric nuclear matter (see, e.g., Ref. [45] for such a procedure in the case of the isoscalar sector).

Shown in Fig. 1 is the density dependence of $E_{\text{sym}}(\rho)$ and its self-energy decomposition according to Eq. (34) for four interactions, i.e., FSUGold [46], IU-FSU [47], NL $\rho\delta$ [48] and HA [49]. FSUGold and IU-FSU are two accurately calibrated interactions based on the ground state properties of closed-shell nuclei, their linear response, and the structure of neutron stars. Since FSUGold and IU-FSU do not consider the isovector-scalar δ meson field, one thus has $\Sigma_S^{\text{sym},1}(\rho) = 0$. On the other hand, both NL $\rho\delta$ and HA include the δ meson field with the latter fitting successfully some results obtained from the microscopic DBHF approach while the former fitting the empirical properties of asymmetric nuclear matter and describing reasonably well the binding energies and charge radii of a large number of nuclei [38]. From Fig. 1, one can see that while the kinetic contribution $E_{\text{sym}}^{\text{kin}}(\rho)$ is roughly the same for different interactions, the different interactions predict significantly different values for $E_{\text{sym}}^{\text{1st},S}(\rho)$ and $E_{\text{sym}}^{\text{1st},V}(\rho)$. In particular, compared with FSUGold

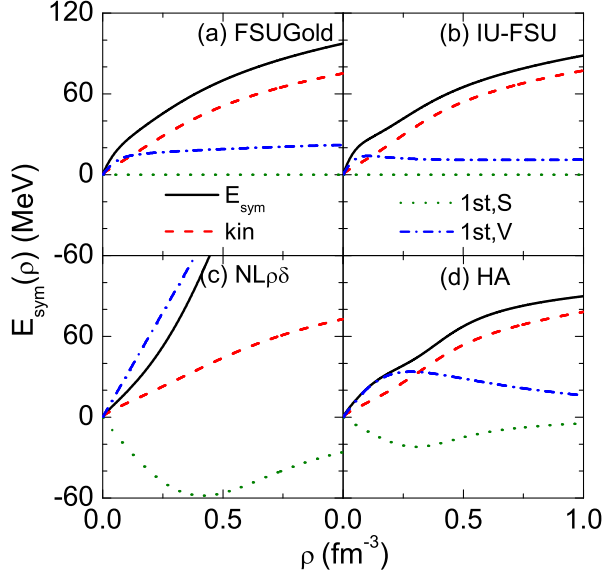


FIG. 1: (Color online) Density dependence of $E_{\text{sym}}(\rho)$ and its self-energy decomposition according to Eq. (34) in the nonlinear RMF model with different interactions.

and IU-FSU, HA predicts very similar total $E_{\text{sym}}(\rho)$ but significantly different $E_{\text{sym}}^{1\text{st},S}(\rho)$ and $E_{\text{sym}}^{1\text{st},V}(\rho)$.

Similarly, we show in Fig. 2 the density dependence of the slope parameter $L(\rho)$ and its self-energy decomposition according to Eq. (37) for FSUGold, IU-FSU, $\text{NL}\rho\delta$ and HA. Again, it is seen that the different interactions predict roughly same kinetic contribution $L^{\text{kin}}(\rho)$ but significantly different values for $L^{1\text{st}}(\rho)$ and $L^{2\text{nd}}(\rho)$. In particular, one can see that the higher-order contribution $L^{2\text{nd}}(\rho)$ from the second-order symmetry self-energies generally cannot be neglected, agreeing well with the recent non-relativistic calculations [24].

IV. SUMMARY AND OUTLOOK

Using the Hugenholtz-Van Hove theorem, we have shown that the symmetry energy $E_{\text{sym}}(\rho)$ and its density slope $L(\rho)$ can be decomposed analytically in terms of the Lorentz covariant nucleon self-energies in asymmetric nuclear matter, and the corresponding expressions have been derived for the first time. These general expressions for the covariant self-energy decomposition of $E_{\text{sym}}(\rho)$ and $L(\rho)$ are useful for determining the density dependence of the symmetry energy, deciphering the Lorentz structure of the symmetry energy, and understanding the microscopic origins of the symmetry energy. As an example, we have analyzed the Lorentz covariant nucleon self-energy decomposition of $E_{\text{sym}}(\rho)$ and $L(\rho)$ within the nonlinear σ - ω - ρ - δ relativistic mean field model and derived the corresponding analytical expressions for $E_{\text{sym}}(\rho)$ and $L(\rho)$, which are potentially useful for fixing the isovector parameters in the RMF model from fitting the empirical properties of asymmetric nuclear matter.

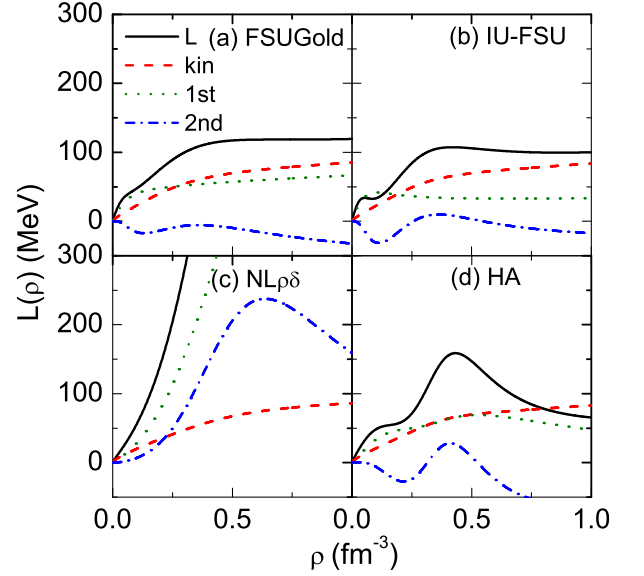


FIG. 2: (Color online) Density dependence of $L(\rho)$ and its self-energy decomposition according to Eq. (37) in the nonlinear RMF model with different interactions.

From analyzing the self-energy decomposition of $E_{\text{sym}}(\rho)$ and $L(\rho)$ within the nonlinear σ - ω - ρ - δ relativistic mean field model, we have found that the results strongly depend on the interactions used and also whether the isovector-scalar δ meson is included or not. These results imply that it is of great importance to determine individually each part of the Lorentz covariant nucleon self-energy decomposition of $E_{\text{sym}}(\rho)$ and $L(\rho)$ from experiments (e.g., Dirac phenomenology) or microscopic calculations based on nucleon-nucleon interactions derived from scattering phase shifts (e.g., DBHF). On the other hand, the Lorentz covariant nucleon self-energies in asymmetric nuclear matter can also be determined from quantum chromodynamics (QCD) by means of QCD sum-rule techniques [50, 51]. The general expressions of the Lorentz covariant nucleon self-energy decomposition of $E_{\text{sym}}(\rho)$ and $L(\rho)$ presented in this work are thus very useful for determining the symmetry energy from QCD. These studies are in progress.

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